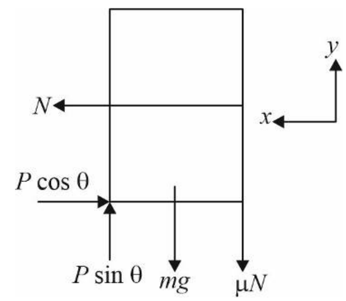
Solutions to JEE Advanced Booster Test – 1 | 2024 | Code A

[PHYSICS]

1.(AD) Impending state of motion is a critical border line between static and dynamic state of body. The block under the influence of $P\sin\theta$ (Component of P) may have a tendency to move upward or it may be assumed that $P\sin\theta$ just prevents downward fall of the block. Therefore there are two possibilities:

Case (I)

Impending motion upward : In this case force of friction is downward from conditions of equilibrium



$$\sum F_x = N - P\cos\theta = 0$$

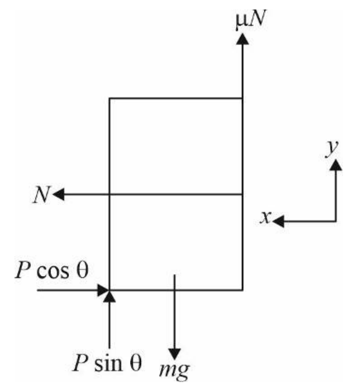
or $$N = P\cos\theta$$

$$\sum F_y = P\sin\theta - \mu N - mg = 0$$

or $$P\sin\theta - \mu P\cos\theta - mg = 0 \quad \text{or} \quad P_{\max} = \frac{mg}{\sin\theta - \mu\cos\theta}$$

Case (II)

Impending motion downward : In this case friction force acts upward



$$\sum F_x = N - P\cos\theta = 0$$

or $$N = P\cos\theta$$

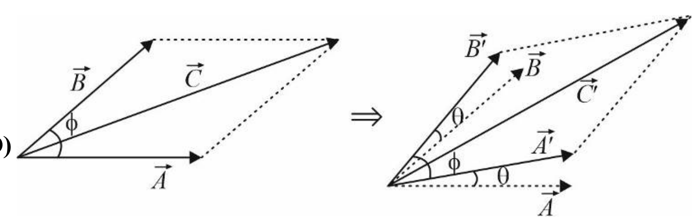
$$\sum F_y = P\sin\theta + \mu N - mg = 0$$

or $$P\sin\theta + \mu P\cos\theta - mg = 0 \quad \text{or} \quad P_{\min} = \frac{mg}{\sin\theta + \mu\cos\theta}$$

Therefore the block will be in state of equilibrium for

$$\frac{mg}{\sin\theta + \mu\cos\theta} \le P \le \frac{mg}{\sin\theta - \mu\cos\theta}$$

2.(BCD)



$$\vec{A} + \vec{B} = \vec{C} \;\&\; \vec{A}' + \vec{B}' = \vec{C}'$$

$$\therefore \quad \vec{A} + \vec{B} \ne \vec{A}' + \vec{B}' \ne \vec{C}$$

$$\vec{A}' \cdot \vec{B}' = \vec{A} \cdot \vec{B}$$

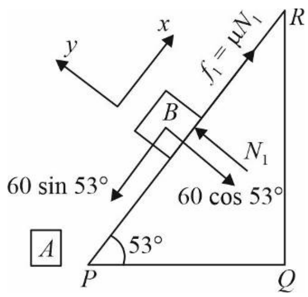
$$|\vec{A}' + \vec{B}'| = |\vec{A} + \vec{B}| = |\vec{C}|$$

3.(AC) When wedge PQR rests on face PQ, block A will slide down and block B will be about to start sliding because for block B,

Angle of repose for block B $(\theta_R) = \tan^{-1}(\mu)$

$$\Rightarrow \quad \theta_R = \tan^{-1}(4/3) = 53^\circ$$

$$\Rightarrow \quad \theta_R = \angle RPQ$$

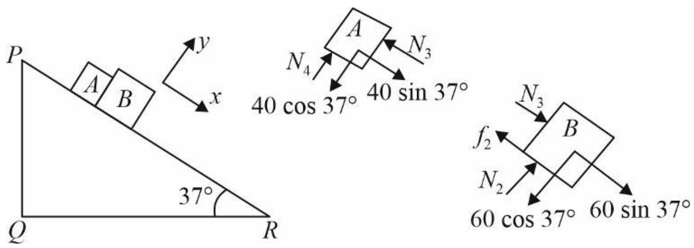


For equilibrium of block *B*

$\sum F_x = 0 \quad \Rightarrow \quad f_1 = 60\sin 53°$

$\Rightarrow \quad f_1 = 48N$

When wedge rests about face QR,



For equilibrium of A:

$$\sum F_x = 0 \Rightarrow \quad 40\sin 37° - N_3 = 0 \quad \Rightarrow N_3 = 24N$$

For equilibrium of B:

$$\sum F_y = 0 \Rightarrow \quad N_2 - 60\cos 37° = 0 \quad \Rightarrow \quad N_2 = 48N$$

So, $\quad (f_2)_{\max} = \mu N_2 = \left(\frac{4}{3}\right)(48) = 64N$

And $\quad \sum F_x = 0 \quad \Rightarrow \quad 60\sin 37° + N_3 - f_2 = 0$

$\Rightarrow \quad 60\sin 37° + 24 = f_2$

$\Rightarrow \quad f_2 = 60N \quad \{f_2 < (f_2)_{\max}\}$

Hence $\quad \dfrac{f_1}{f_2} = \dfrac{48}{60} = \dfrac{4}{5}$

4.(AB) Component of $\vec{A}$ along $\vec{B}$ is $|\vec{A}|\cos\theta\hat{B}$ for θ being the angle between the vectors.

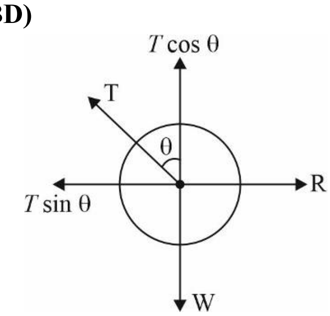
Also $\hat{B} = \dfrac{\hat{i}+\hat{j}}{\sqrt{2}}$. So, choice (a) is correct.

The vector $(\hat{i}-\hat{j})$ is perpendicular to the vector $(\hat{i}+\hat{j})$.

So, the other resolved component is $|\vec{A}|\sin\theta\left(\dfrac{\hat{i}-\hat{j}}{\sqrt{2}}\right)$.

5.(BC) The resultant of three vectors is zero only if they can form a triangle. But three vectors lying in different planes cannot form a triangle.

6.(ABD)



$T^2 = R^2 + W^2$

$\sum \vec{F}_{net} = 0$

So, $\vec{T} + \vec{W} + \vec{R} = 0$

7.(C) & 8.(C)

$\vec{\tau}_P = \vec{r}_{OP} \times \vec{F}$

$= (xi + yj - \hat{k}) \times (2i - j + \hat{k})$

$\vec{\tau}_P = (y-1)i - (x+2)j - (x+2y)\hat{k}$

Given, $\vec{\tau}_P = -4\hat{j} - 4\hat{k}$

$\therefore$ $x = 2$ and $y = 1$

So, coordinates of P are $(2, 1, -1)$.

Since x-component of torque at Q about 'O' is zero and y and z-components are equal in magnitude and directed along the negative directions of the respective axes, $\vec{\tau}_Q \uparrow\uparrow \vec{\tau}_P$

$\Rightarrow$ $\vec{\tau}_Q = \lambda \vec{\tau}_P = \lambda(-4\hat{j} - 4\hat{k})$ where λ is a constant

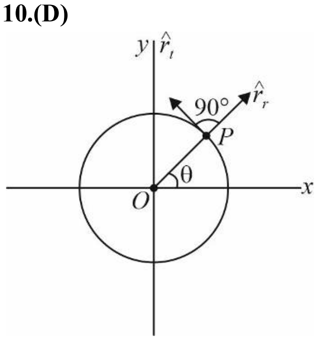
Also $|\vec{\tau}_Q| = 10\sqrt{2}$

$\Rightarrow$ $\sqrt{(4\lambda)^2 + (4\lambda)^2} = 10\sqrt{2}$

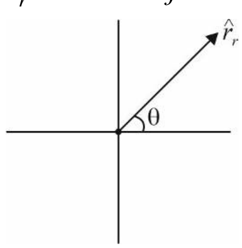
$\Rightarrow$ $32\lambda^2 = 200$ $\Rightarrow$ $\lambda = \frac{5}{2}$

So, $\vec{\tau}_Q = \frac{5}{2}(-4\hat{j} - 4\hat{k})$ $\Rightarrow$ $\vec{\tau}_Q = -10\hat{j} - 10\hat{k}$ N-m

9.(B) & 10.(D)



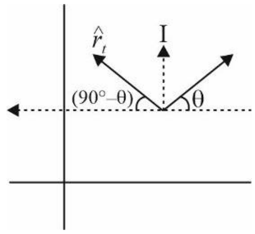
9. $\vec{r}_r = \hat{i}\cos\theta + \hat{j}\sin\theta$



10. $\vec{r}_t = -\hat{i}\cos\theta' + \hat{j}\sin\theta'$

$= -\hat{i}\cos(90° - \theta) + \hat{j}\sin(90° - \theta)$

$= -\hat{i}\sin\theta + \hat{j}\cos\theta$



11.(D) $|\vec{A}| = 4; |\vec{B}| = 5; |\vec{C}| = 3$

(A) $|\vec{A} - \vec{B}| \le (4+5)$

(B) If $\vec{A}, \vec{B}, -\vec{C}$ form a right angle triangle then $\vec{A} + \vec{B} - \vec{C} = 0$

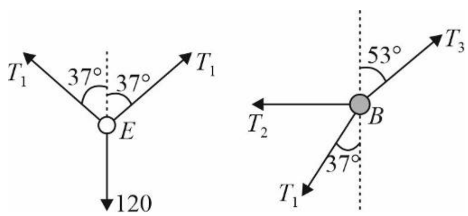
(C) $\vec{A} \cdot (\vec{B} - \vec{C}) = |\vec{A}||\vec{B} - \vec{C}|\cos\theta = 4.8 \cdot 1 = 32$

(D) $|\vec{A} + \vec{B} - \vec{C}| \le (4+5+3)$

12.(A) From figure,

$$2T_1 \cos 37° = 120$$

$$\Rightarrow \quad 2T_1 \times \frac{4}{5} = 120 \Rightarrow T_1 = 75N$$



$$T_1 \cos 37° = T_3 \cos 53°$$

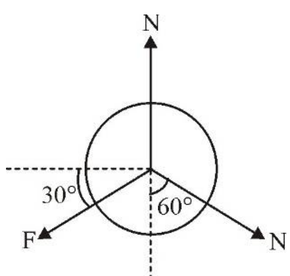
$$\Rightarrow \quad 75 \times \frac{4}{5} = T_3 \times \frac{3}{5} \Rightarrow \quad T_3 = 100N$$

$$T_2 + T_1 \sin 37° = T_3 \sin 53°$$

$$\Rightarrow \quad T_2 + 75 \times \frac{3}{5} = 100 \times \frac{4}{5} \Rightarrow \quad T_2 = 35N$$

SECTION 2

1.(1)



$$N = F \sin 30° + N \cos 60°$$

$$F \cos 30° = N \sin 60°$$

Solve to get

$$F = N$$

2.(1.50) Using the parallelogram law of vector addition,

$$R_1 = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 60°} = \sqrt{F_1^2 + F_2^2 + F_1F_2}$$

$$R_2 = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 120°} = \sqrt{F_1^2 + F_2^2 - F_1F_2}$$

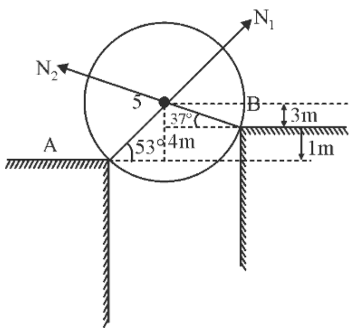
Therefore, $\dfrac{F_1^2+F_2^2+F_1F_2}{F_1^2+F_2^2-F_1F_2}=\left(\dfrac{R_1}{R_2}\right)^2=\dfrac{19}{7}$ $\Rightarrow$ $\dfrac{\left(\dfrac{F_1}{F_2}\right)^2+1+\dfrac{F_1}{F_2}}{\left(\dfrac{F_1}{F_2}\right)^2+1-\dfrac{F_1}{F_2}}=\dfrac{19}{7}$

Now, let $\dfrac{F_1}{F_2}=K$

Therefore, $\dfrac{K^2+K+1}{K^2-K+1}=\dfrac{19}{7}$ $\Rightarrow$ $12K^2-26K+12=0$

Solving, we get $K=\dfrac{3}{2}$ or $\dfrac{2}{3}$ $\therefore$ $F_1>F_2\Rightarrow K>1\ \therefore K=\dfrac{3}{2}=1.5$

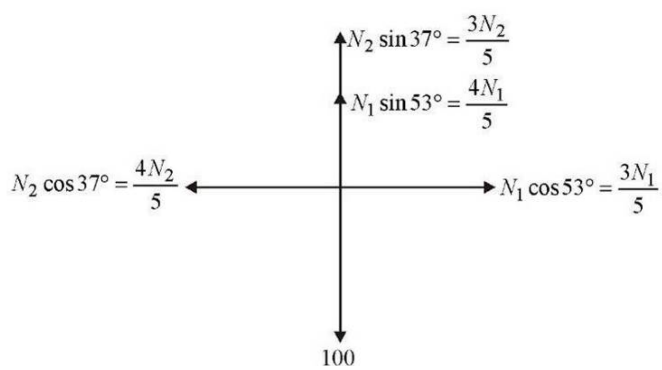
3.(32) Let N_1 and N_2 denote the forces acting on the sphere from the supports *A* and *B* respectively



Balancing forces,

$$\frac{3N_1}{5}=\frac{4N_2}{5} \quad \text{…(i)}$$

$$\frac{4N_1}{5}+\frac{3N_2}{5}=40 \quad \text{…(ii)}$$



Solving, we get

$$N_1=32\ N$$

$$N_2=24\ N$$

4.(10) Let East be $+x$ direction, and North be $+y$ direction.
Let initial position of *B* be the origin

Final position of *B*, $\bar{x}_B=4\hat{i}+(-2\hat{i}-2j)=2\hat{i}-2\hat{j}$

Initial position of A is $10\hat{i}$ $\Rightarrow$ Final position of A, $\bar{x}_A=10\hat{i}+(6\hat{i}+6\hat{j})+(-8\hat{i})=8\hat{i}+6\hat{j}$

Final distance between *A* and *B* is $|\bar{x}_A-\bar{x}_B|=|6\hat{i}+8\hat{j}|=\sqrt{36+64}=10m$

5.(3) $\vec{P}\times\vec{R}=2\hat{i}-2\hat{j}+\hat{k}$

$|\vec{P}\times\vec{R}|=3$

$|\vec{Q}-\vec{P}|=2\sqrt{2}$

$(\vec{Q}-\vec{P})^2=8$

$Q^2+P^2-2\vec{Q}\cdot\vec{P}=8$

$Q^2+P^2-2Q=8$

$Q^2 + 9 - 2Q = 8$

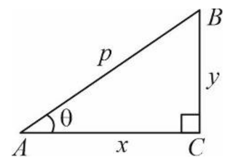
$|Q| = 1$

$|(\vec{P} \times \vec{R}) \times \vec{Q}| = |\vec{P} \times \vec{R}||\vec{Q}| \sin 30°$

$= 3 \times (1)\frac{1}{2} = \frac{3}{2}$

$n = 3$

6.(1) $\overrightarrow{AB} \cdot \overrightarrow{AC} + \overrightarrow{BC} \cdot \overrightarrow{BA} + \overrightarrow{CA} \cdot \overrightarrow{CB} = mp^2$



$p \cos \theta x + p \sin \theta y + 0 = mp^2$

$p \cos \theta \cdot p \cos \theta + p \sin \theta \cdot p \sin \theta = mp^2$

$p^2 (\cos^2 \theta + \sin^2 \theta) = mp^2$

$m = 1$

[CHEMISTRY]

1.(ABC)

$$\underset{(n=5)}{2H_3AsO_4} + \underset{(n=2)}{5H_2S} \longrightarrow \underset{(n=10)}{As_2S_5} + \underset{(n=10/8)}{8H_2O}$$

Mole of H_3AsO_4 taken $= \frac{35.5}{142} = 0.25$

Mole of As_2S_5 formed $= \frac{1}{2} \times$ mole of H_3AsO_4 taken $= \frac{1}{2} \times 0.25$

Mole of H_2O formed $= 4 \times$ mole of H_3AsO_4 taken $= 0.25 \times 4 = 1$

Equivalent mass of $H_3AsO_4 = \frac{142}{5} = 28.4$

Equivalent of H_3AsO_4 used $=$ Equivalent of H_2S used $= 0.25 \times 5 = 1.25$

Hence (A), (B) and (C) are correct.

2.(ABD)

2 mole equimolar mixture contain one mole each of Na_2CO_3 and $NaHCO_3$.

$Na_2CO_3 \xrightarrow{\Delta}$ No effect

$2NaHCO_3 \xrightarrow{\Delta} Na_2CO_3 + \underbrace{H_2O + CO_2}_{\text{Loss in mass}}$

Mole of H_2O formed $= 0.5$

Mass of H_2O formed $= 0.5 \times 18 = 9$ gm

Mole of CO_2 formed $= 0.5$

Mass of CO_2 formed $= 0.5 \times 44 = 22$ gm

Total loss in mass $= 9 + 22 = 31$ gm

Mass of residue $= \underbrace{w_{Na_2CO_3}}_{\text{takan initially}} + \underbrace{w_{Na_2CO_3}}_{\text{formed from } NaHCO_3} = (1 \times 106) + (0.5 \times 106) = 159$ gm

Equivalent of $Na_2CO_3 =$ Equivalent of HCl

$\frac{159}{106} \times 2 = 1 \times V \quad \Rightarrow \quad V = 3L$

Equivalent of $HCl = \frac{1}{2}$ Equivalent of Na_2CO_3

$1 \times V = \frac{1}{2} \times 1 \times 2 \quad \Rightarrow \quad V = 1L$

Hence (A), (B) and (D) are correct.

3.(ABD)

	2A	+ 3B	$\longrightarrow$ 4C	+ 5D	
	1 mole	1 mole	0	0	(B is limiting reagent)
	$\left(1-\frac{2}{3}\right)$	0	$\frac{4}{3}$	$\frac{5}{3}$	(Total moles $= \frac{10}{3}$)
	$\frac{1}{3}$	0	$\frac{4}{3}$	$\frac{5}{3}$	
χ	0.1	0	0.4	0.5	

Hence (A), (B) and (D) are correct.

4.(ACD)

$H_3PO_2 (n = 1) \quad : \quad E = M^\circ$

$H_3PO_3 (n = 2) \quad : \quad E = \frac{M^\circ}{2}$

$H_2C_2O_4 (n = 1) \quad : \quad E = M^\circ$

$H_3PO_4 (n = 1) \quad : \quad E = M^\circ$

Hence (A), (C) and (D) are correct.

5.(ABC)

Strength of Na_2CO_3 solution $= \frac{0.1}{10} \times 1000 = 10 g/L$

Molarity of Na_2CO_3 solution $= \frac{10}{M_0} = \frac{10}{286}$

Normality of Na_2CO_3 solution $= \frac{10}{286} \times 2$

Equivalent of H^+ ions (in 30 mL) = Equivalent of Na_2CO_3 solution

$$N \times \frac{30}{1000} = \frac{10}{286} \times 2 \times \frac{42.9}{1000}$$

$$N_{H^+} = \frac{10 \times 2 \times 42.9}{286 \times 30} = 0.1$$

$$0.1 \times 2000 = (16 \times 5) + (10 \times 4.8) + (10 \times 3 \times x)$$

$$200 = 80 + 48 + 30x \quad \Rightarrow \quad x = 2.4 mL$$

Mole of PO_4^{3-} = Mole of $H_3PO_4 = 10 \times \frac{2.4}{1000} = 0.024$

Mass of $PO_4^{3-} = 0.024 \times 95 = 2.28 gm$

6.(AC)

Chemical interaction between A and B_2 results in formation of either AB_2 or A_2B_2 in a reaction, not both in the same reaction. In this case there are two parallel reactions taking place for formation of both AB_2 and A_2B_2.

Reaction-I : $A + B_2 \longrightarrow AB_2$

Reaction-II : $2A + B_2 \longrightarrow A_2B_2$

Mole of A taken $= \frac{w}{200}$

Mole of B taken $= \frac{w}{254}$

Mole of A taken > Mole of B_2 taken

Let x mole of A are used in reaction-I while 2y mole are used for reaction-II.

Mole of A $= x + 2y$

Mole of B_2 used in reaction-I are x and y mole are used in reaction-II.

Moles of B_2 used $= x + y$.

According to question A and B_2 are used completely hence

$$x + 2y = \frac{w}{200} \qquad \text{...... (1)}$$

$$x + y = \frac{w}{254} \qquad \text{...... (2)}$$

On solving $\quad y = \frac{w \times 54}{200 \times 254} =$ mole of A_2B_2 formed

$x = \frac{146w}{200 \times 254} =$ mole of AB_2 formed

Mole of AB_2 formed > Mole of A_2B_2 formed

7.(A) Electrolysis of water, atom economy $= \frac{32}{36} \times 100 = 88.9\%$

Catalytic decomposition of H_2O_2, atom economy $= \frac{32}{68} \times 100 = 47.1\%$

Producing oxygen by the electrolysis of water has the better atom economy and produce less waste.

8.(B) **I** $\Rightarrow$ Atom economy $= 49.8\%$; Percentage yield $= 75\%$

II $\Rightarrow$ Atom economy $= 49\%$; Percentage yield $= 64\%$

9-10 **9.(C)** **10.(B)**

Volume of oil = 5 mL

Mass of oil $= 5 \times 0.95 = 4.75$ gm

Mole of oil $= \frac{4.75}{240} = 1.98 \times 10^{-2}$

Volume of one molecule $= a^3$

Number of molecules $= \frac{5}{a^3} \quad \Rightarrow \quad a = \frac{5}{2 \times 10^7} = 2.5 \times 10^{-7}$

$$= \frac{5}{(2.5 \times 10^{-7})^3} = 3.2 \times 10^{20}$$

11.(C) **(A)** $\frac{W_{H_3PO_3}}{82} \times 2 = 1.5 \times 0.4 \quad \Rightarrow \quad W_{H_3PO_3} = 24.6$ gm

(B) $\frac{W_{MgCO_3}}{84} = 0.2 \quad \Rightarrow \quad W_{MgCO_3} = 16.8$ gm; 16.8% pure

(C) Mole of Ti taken = Mole of Ti in $Ti_{1.44}O_1$

$$\frac{1.44}{48} = \left(\frac{x}{(1.44 \times 48) + 16}\right) \times 1.44 \quad \Rightarrow \quad x = 1.77 \text{ gm}$$

(D) $\left(\frac{(1.84-x)}{84}\right)+\left(\frac{x}{100}\right)=\frac{0.88}{44}$ $\Rightarrow$ $x = 1gm$ $[W_{CO_2} = 1.84 - 0.96 = 0.88\,g]$

12.(C) **(P)** HCl reacts with NaOH

Mole of HCl $= \frac{5.5}{36.5} =$ Mole of Cl^-

Mole of NaOH $= \frac{5.5}{40} =$ Mole of Na^+

Solution is acidic, $[Cl^-] = \frac{5.5}{36.5} \times \frac{1000}{200} = 0.75\,M$

$[Na^+] = \frac{5.5}{40} \times \frac{1000}{200} = 0.69\,M$

(Q) No reaction between HCl and CH_3COOH but mixing results in dilution.

Solution is acidic $[HCl] = [Cl^-] = 0.15\,M$

(R) NaOH reacts with HCl

Mole of NaOH $=$ Mole of $Na^+ = \frac{6}{40} = 0.15$

Mole of HCl $=$ Mole of $Cl^- = 0.15$

Solution is neutral, $[Na^+] = [Cl^-] = 0.15\,M$

(S) NaCl reacts with $AgNO_3$ and form insoluble AgCl.

Mole of NaCl $=$ Mole of $Na^+ = 0.15$

Mole of AgCl precipitate $= 0.15$

Solution is neutral, $[Na^+] = 0.15\,M, [Cl^-] = 0\,M$

SECTION 2

1.(2.50) Hard water contain $CaSO_4$ and $Ca(HCO_3)_2$.

Mole of Ca^{2+} = Mole of $SO_4^{2-} = \frac{96}{96} = 1$ (in 10^6 mL solution)

Mole of $Ca^{2+} = \frac{1}{2} \times$ Mole of $HCO_3^- = \frac{183}{61} \times \frac{1}{2} = 1.5$ (in 10^6 mL solution)

Total moles of $Ca^{2+} = 1 + 1.5 = 2.5$

Molarity $= \frac{2.5}{10^6} \times 10^3 = 2.5 \times 10^{-3} = x \times 10^{-3}$ $\Rightarrow$ $x = 2.5$

2.(696)

Mass of Pd taken $=$ w gm

Volume of Pd $= \frac{w}{12}$ cc (at STP)

Volume of $H_2 = \frac{w}{12} \times 936$ cc

Mole of H $= 2 \times \frac{w}{12} \times \frac{936}{22400}$

Molarity of H $= \frac{2 \times w \times 936 \times 1000}{12 \times 22400 \times w} = 6.96 = 696 \times 10^{-2}$ $\Rightarrow$ $x = 696$ (round off value)

3.(3.60)

Molar mass of $CaHAsO_4 \cdot 2H_2O = 216\,g / mole$

Mole of O atoms $= 6 \times$ Mole of $CaHAsO_4 \cdot 2H_2O$

Number of O atoms $=$ Mole of $O \times 6 \times 10^{23}$

$$P \times 10^{22} = 6 \times 10^{23} \times 6 \times \frac{2.16}{216} \quad \Rightarrow \quad P = 3.6$$

4.(10) $M = \frac{10xd}{M_0} = \frac{10 \times 30.7 \times 1.11}{34} = x\,M \quad \Rightarrow \quad x = 10.02 \approx 10$

5.(37) Potassium sulphide is K_2S and potassium chloride is KCl.

Let mass of K_2S be w gm.

$$\left(\frac{2 \times w}{112}\right) + \left(\frac{5 - x}{75}\right) = \frac{3}{40} \quad \Rightarrow \quad x = \frac{w}{5} \times 100 = 36.84 \approx 37$$

6.(0.20)

$$NaOH + Na_2CO_3 \cdot NaHCO_3 \cdot 2H_2O \longrightarrow 4Na^+ + 2CO_3^{2-} + 3H_2O$$

$[HCO_3^-] = (y - x) = 0.10$

$[Na^+] = 4x + 3(y - x) = 0.50$

$4x + 0.30 = 0.50$

$x = \frac{0.50 - 0.30}{4} = \frac{0.20}{4} = 0.05$

$y = 0.10 + 0.05 = 0.15$

$x = 0.05;\ y = 0.15$

[MATHEMATICS]

1.(ABCD)

$(\log_2 x)^2 - 4\log_2 x - 12 = (m+1)^2$

$t^2 - 4t - \left\{12 + (m+1)^2\right\} = 0$

$$t = \frac{4 \pm \sqrt{16 + 4\left(12 + (m+1)^2\right)}}{2}$$

$D > 0 \quad \Rightarrow \quad$ (A) is correct

Now $D_{\min}$ when $m = -1$

$\log_2 x = \frac{4 \pm 8}{2} = 6$ or -2

$x = 2^6$ or 2^{-2}

$\Rightarrow$ (C) and (D) are correct

Also $\log_2 x_1 + \log_2 x_2 = 4$

$\log_2 x_1 x_2 = 4$

$\Rightarrow \quad x_1 x_2 = 2^4$

$\Rightarrow$ (B) is correct

2.(ABC)

$$f(x) = Ax^2 + Bx + C$$

$$A = a + b - 2c = (a-c) + (b-c) > 0$$

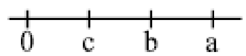
$\Rightarrow \quad A > 0$

$\Rightarrow \quad$ mouth opens upwards

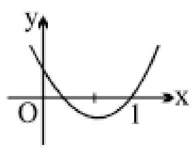
Now $x = 1$ is obvious solution therefore both roots are rational.

$$\underbrace{(b-a)}_{-ve} + \underbrace{(c-a)}_{-ve} < 0$$

$\Rightarrow \quad B < 0;$



Vertex $= -\dfrac{B}{2A} > 0$



hence abscissa $'a'$ of the vertex > 0

(D) need not be correct as with

$a = 5, b = 4, c = 2, P < 0$

And $\quad a = 6, b = 3, c = 2, P > 0$

$\Rightarrow \quad$ (A), (B) and (C) are correct.

3.(ABC)

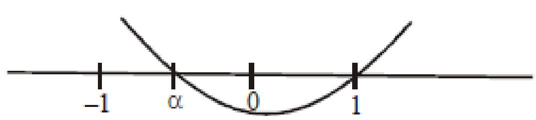
$$f(x) = (p + q - 2r)x^2 + (q + r - 2p)x + (r + p - 2q)$$

Given $p > 0; q > 0; r > 0$ and

$f(x)$ has a root in $(-1, 0)$

Also $f(1) = 0$

$\therefore \quad$ one root of $f(x)$ is 1



$\Rightarrow \quad f(0) < 0$

$\therefore \quad r + p - 2q < 0$

$$\frac{r+p}{q} < 2$$

Also product of roots $= \dfrac{r + p - 2q}{p + q - 2r}$

$\alpha \cdot 1 = \dfrac{r + p - 2q}{p + q - 2r} \qquad$ (since p, q, r are rational)

$\Rightarrow$ α is rational have both roots are rational

Also $r + p < 2q$

$(r+p)^2 < 4q^2$

$4q^2 > (p+r)^2$

$$\underbrace{4q^2 - 4pr}_{\substack{\text{discriminant of} \\ px^2+2qx+r}} > \underbrace{(p-r)^2}_{\text{a } +ve \text{ quantity}}$$

$\Rightarrow$ roots of $px^2 + 2qx + r = 0$ are real and distinct

4.(CD) $(a-1)\left(x^2+\sqrt{3}x+1\right)^2 - (a+1)\left[\left(x^2+1\right)^2 - \left(x\sqrt{3}\right)^2\right] \le 0$

or $(a-1)\left(x^2+\sqrt{3}x+1\right)^2 - (a+1)\left[x^2+x\sqrt{3}+1\right)\left(x^2-x\sqrt{3}-1\right) \le 0$

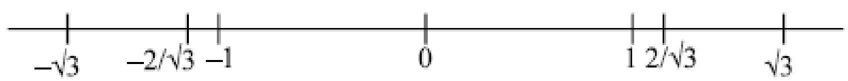
$\left(x^2+\sqrt{3}x+1\right)\left[(a-1)\left(x^2+\sqrt{3}x+1\right)-(a+1)\left(x^2-\sqrt{3}x+1\right)\right] \le 0 \ \forall x \in R$

$\Rightarrow$ $-2\left(x^2+1\right)+2a\sqrt{3}x \le 0$

$\Rightarrow$ $x^2 - a\sqrt{3}x + 1 \ge 0 \ \forall x \in R$

$\Rightarrow$ $3a^2 - 4 \le 0 \quad (D \le 0)$

$\Rightarrow$ $a \in \left[-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right]$



$\Rightarrow$ number of possible integral value of 'a' is $\{-1,0,1\}$

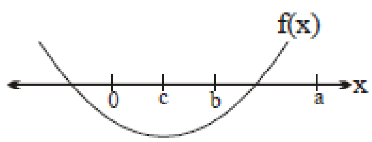
$\Rightarrow$ 3

and sum of all integral values of 'a' is $-1+0+1=0$

5.(ABC)

Let $f(x) = a(x-b)(x+c) + b(x-a)(x+c) - c(x-a)(x-b)$

Given a, b, c are sides of triangle, hence they are all positive.



Now, $f(a) = a(a-b)(a+c) > 0$

$f(b) = b(b-a)(b+c) < 0$

$$f(c) = a\underbrace{(c-b)}_{-ve}\underbrace{(c+c)}_{+ve} + b\underbrace{(c-a)}_{-ve}\underbrace{(c+c)}_{+ve} - c\underbrace{(c-a)}_{-ve}\underbrace{(c-b)}_{-ve} < 0$$

$f(0) = -abc - abc - abc = -3abc < 0.$

As $f(0) < 0$, hence one root is positive and one root is negative $\Rightarrow$ roots are of opposite signs.

Hence $f(x) = 0$ has real and distinct roots and positive roots lies in (b, a).

6.(BC)

(A) If for some value of x say $x = x_1$, the equation $E = 0$ has one root infinite and other root a non zero finite,

so $u = 0$ and $v \neq 0$

$\therefore$ $u = 0$ and $v = 0$ do not have a common root.

$\Rightarrow$ **(A)** is False

(B) If for some value of x say $x = x_2$, the equation $E = 0$ has both root infinite roots,

so $u = 0$ and $v = 0$

$\therefore$ $u = 0$ and $v = 0$ will have a common root.

$\Rightarrow$ **(B)** is True

(C) If for some value of x say $x = x_3$, the equation $E = 0$ becomes an identity in y,

so $u = 0, v = 0$ and $w = 0$

$\therefore$ $u = 0, v = 0$ and $w = 0$ must have a root, common to all of them.

$\Rightarrow$ **(C)** is True

(D) If for some value of x say $x = x_4$, the equation $E = 0$ has both roots real and distinct then $v = 0$ and $w = 0$ must have a common root is not necessary.

$\Rightarrow$ **(D)** is False

Paragraph for Q.7(A)-8(D)

(i) We have $g(x) = x^2 - (m+1)x + m - 1$

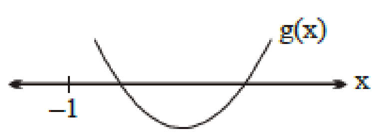
Now,

discriminant $= (m+1)^2 - 4(m-1)$

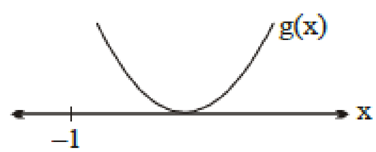
$= m^2 - 2m + 5$, which is always positive.

$\Rightarrow$ $g(x)$ always positive is not possible for any integral value of m.

(ii) Different possibilities are as follows : -



or



If both roots of $g(x) = 0$ are greater than –1 then 3 conditions should be satisfied simultaneously.

(1) $D \geq 0$ (2) $\frac{-B}{2A} > -1$ (3) $g(-1) > 0$

Now, (1) $\Rightarrow$ $(m+1)^2 - 4(m-1) \geq 0$

$\Rightarrow$ $m^2 - 2m + 5 \geq 0, \quad \forall m \in R$.

(2) $\Rightarrow$ $\frac{m+1}{2} > -1$

$\Rightarrow \quad m > -3$

and (3) $\Rightarrow \quad 1+(m+1)+m-1>0$

$\Rightarrow \quad m > -\frac{1}{2}$

Hence from (1), (2) & (3) we get $m \in \left(-\frac{1}{2}, \infty\right)$

Paragraph for Q.9(D) – 10(B)

(i) For roots of $f(x)=0$ to be equal in magnitude and opposite in sign

Sum of the roots = 0 and product of the roots < 0

Sum of the roots = 0

$\Rightarrow \quad a = 0 \quad \ldots(1)$

Product of the roots < 0

$\Rightarrow \quad 5a^2 - 6a < 0, a \in \left(0, \frac{6}{5}\right) \quad \ldots(2)$

$(1) \cap (2) \quad \Rightarrow \quad a \in \phi$

(ii) Range of $f(x)$ is $[-5, \infty)$ for every real x

or range $f(x)+5$ is $[0, \infty)$ for every real x

Now $f(x)+5 = x^2 - 4ax + 5a^2 - 6a + 5$

for range to be $[0, \infty)$, $D = 0$

$\Rightarrow \quad 16a^2 - 4\left(5a^2 - 6a + 5\right) = 0$

$\Rightarrow \quad a^2 - 6a + 5 = 0 \quad \Rightarrow \quad a = 5$ or 1

11.(A) **A-**Q; **B-**Q, R, S; **C-**R,S

(A) Let α be a common root.

then $\alpha^3 + K\alpha^2 + 3 = 0 \quad \ldots(1)$

and $\alpha^2 + K\alpha + 3 = 0 \quad \ldots(2)$

Now,

$(1) - \alpha \times (2) \Rightarrow 3 - 3\alpha = 0$

$\therefore \alpha = 1$

So, from (1), wet $1 + k + 3 = 0$

$\therefore k = -4$

(B) Let $f(x) = 5x^2 + (k+1)x + k$

Now, $f(1)f(3) < 0$

$\Rightarrow \quad (2k+6)(4k+48) < 0$

$\Rightarrow \quad -12 < k < -3$

Also, checked at end points.

For $k=-3$,

We get $5x^2-2x-3=0 \left\langle \begin{matrix} 1 \\ \frac{-3}{5} \end{matrix} \right.$

And for $k=-12$,

We get $5x^2-11x-12=0 \left\langle \begin{matrix} 3 \\ \frac{-4}{5} \end{matrix} \right.$

(C) Let $x^2+13x+44=K^2$

$x^2+13x+\left(44-K^2\right)=0$

$\therefore \quad x=\dfrac{-13\pm\sqrt{169-4\left(44-K^2\right)}}{2}=\dfrac{13}{2}\pm\dfrac{\sqrt{4K^2-7}}{2}$

For x to be an integer $4K^2-7$ must be square of an odd integer.

$\therefore \quad 4K^2-7=(2n+1)^2$

$(2K+2n+1)(2K-2n-1)=7=7\times1$

$\Rightarrow \quad 2K+2n+1=7$ and $2K-2n-1=1$

$\Rightarrow \quad K=2$

Hence $x^2+13x+44=4$

$x^2+13x+40=0$

$(x+8)(x+5)=0$

$\Rightarrow \quad x=-8$ or -5

12.(C) **(A-**Q, R, S; **B-**R, S; **C-**P, Q, R; **D-**P, Q, S**)**

(A) Let $y=|x|$

$x^2-|x|+a=0 \qquad \ldots(1)$

$y^2-y+a=0 \qquad \ldots(2)$

If both roots of (2) are positive then (1) have four solutions. If one root of (2) is positive then (1) have two solutions

and if $a=0$.

$x^2-|x|=0$ has $x=-1,0,1$ as solutions.

(B) $y=\dfrac{(x-4)(x-2)}{(x-2)(x-1)}=\dfrac{x^2-6x+8}{x^2-3x+2}$

$y\neq1$ as when $y=1, x\to\infty$

$y\neq2$ as this is the value when $x\to2$

(C) $x>0,\ \dfrac{1}{2}\log_2 x-2\left(\dfrac{\log_2 x}{2}\right)^2+1>0$

$\Rightarrow \quad \log_2 x-(\log_2 x)^2+2>0$

$\Rightarrow \quad (\log_2 x)^2 - \log_2 x - 2 < 0$

Let $\quad \log_2 x = t$

$t^2 - t - 2 < 0$

$\Rightarrow \quad (t-2)(t+1) < 0$

$\Rightarrow \quad -1 < t < 2$

$\Rightarrow \quad -1 < \log_2 x < 2$

$\Rightarrow \quad \frac{1}{2} < x < 4$

Hence number of integers $\{1, 2, 3\}$

(D) Domain, $x > 1$

$\log_2 \frac{(x-1)(x+2)}{(3x-1)} < 1$

$\frac{(x-1)(x+2)}{(3x-1)} < 2$

Since $x > 1$; $\quad \therefore \quad D^r$ is +ve

$\Rightarrow \quad (x-1)(x+2) < 6x - 2$

$x^2 + x - 2 < 6x - 2$

$x^2 - 5x < 0$

$x(x-5) < 0$

Since $\quad x > 1$

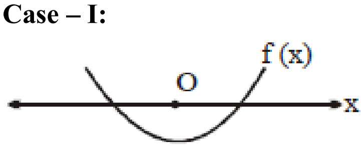
$\therefore \quad (1, 5) \quad \Rightarrow \quad a_1 = 1$ and $a_2 = 5$

$\Rightarrow \quad a_2 - a_1 = 4$ which is divisible by 1, 2, 4

SECTION 2

1.(5) Let $f(x) = x^2 - 2(a+1)x + 3(a-3)(a+1)$

Case – I:

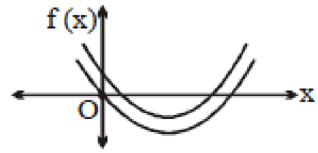


(i)

$f(0) < 0 \quad \Rightarrow (a-3)(a+1) < 0$

$\Rightarrow -1 < a < 3$

Case – II:



(i) $f(0) \geq 0 \quad \Rightarrow (a-3)(a+1) \geq 0$

$\Rightarrow a \in (-\infty, -1] \cup [3, \infty)$

(ii) $D > 0$

$4\left[(a+1)^2 - 3(a-3)(a+1)\right] > 0$

$a^2 + 2a + 1 - 3\left(a^2 - 2a - 3\right) > 0$

$-2a^2 + 8a + 10 > 0$

$a^2 - 4a - 5 < 0$

$(a-5)(a+1) < 0$

$\therefore \quad a \in (-1, 5)$

(iii) $\frac{-B}{2A} > 0$

$(a+1) > 0$

$\therefore \quad a > -1$

$\therefore$ from (i) $\cap$ (ii) $\cap$ (iii) $\cap$ we get $a \in [3, 5)$

Case-I $\cup$ Case-II

$\Rightarrow \quad a \in (-1, 3) \cup [3, 5)$

So, there are 5 possible integral values of a i.e., $a = 0, 1, 2, 3, 4$. **Ans.**

2.(7) $f(x) = (a+2)(a-1)x^2 - (a+5)x - 2 \leq 0$

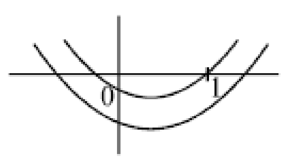
For $a = -2$;

$f(x) = -3x - 2$ which is negative in $[0, 1]$

For $a = 1$;

$f(x) = -6x - 2$ which is negative in $[0, 1]$

Now **Case – I**:



For $a^2 + a - 2 > 0$

i.e. $a > 1$ or $a < -2$ …(1)

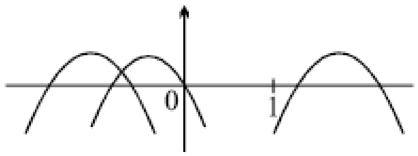
(i) $f(0) \leq 0$ always true

(ii) $f(1) \leq 0$

$\Rightarrow a \in [-3, 3]$ …(2)

From (1) and (2) $a \in [-3, -2) \cup (1, 3]$

Case – II:



$a^2 + a - 2 < 0$

i.e. $-2 < a < 1$

In this case :

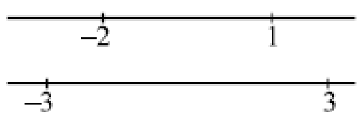
(i) $D \geq 0 \Rightarrow (a+2)^2 \geq 0 \Rightarrow a \in R$

(ii) $f(1) < 0$ & $f(0) \leq 0$ (always true)

$\Rightarrow a \in (-3, 3)$

Common solution $(-2, 1)$

Case – III:



$a < 0$ & $D < 0$

Since $D \geq 0$ $\Rightarrow$ no soution in this case combining all $a \in [-3, 3]$

3.(8) We have $x^2 - 4x - c \geq 0$

$\Rightarrow D \leq 0$

$\therefore 16 + 4c \leq 0$

$\Rightarrow 4c \leq -16$

$\Rightarrow c \leq -4$

Let $f(x) = x^2 - 4x - c$

$\therefore x^2 - 4x - c - \sqrt{8x^2 - 32x - 8c} = 0$

$\Rightarrow f(x) - \sqrt{8f(x)} = 0$

$\therefore \sqrt{f(x)}\left(\sqrt{f(x)} - \sqrt{8}\right) = 0$

$\therefore f(x) = 0$ or $f(x) = 8$

Case – I:

$f(x) = 0$ has two distinct real solutions and $f(x) = 8$ has no real solution.

$\therefore$ For $f(x) = 0, D > 0$ and for $f(x) = 8, D < 0$

$\Rightarrow 16 + 4c > 0$ and $16 + 4(c+8) < 0$

$\Rightarrow c > -4$ and $c < -12$

$\because$ No common solution for 'c' exist.

$\therefore c \in \phi$

Case – II:

$f(x) = 0$ has no real roots but $f(x) = 8$ has two distinct real roots

$\therefore$ For $f(x) = 0, D < 0$ and for $f(x) = 8, D > 0$

$\Rightarrow \quad 16+4c<0$ and $16+4(c+8)>0$

$\Rightarrow \quad c<-4$ and $c>-12$

$\therefore \quad c \in (-12,-4)$

Hence $b-a=-4-(-12)=-4+12=8$

4.(3) Given $\underset{\alpha \;\; \beta=1+\alpha}{4x^2} -(5p+1)x+5p=0$

Clearly, sum of roots $=\alpha+(1+\alpha)=\dfrac{5p+1}{4}$

$$\Rightarrow \quad \alpha=\frac{\left(\frac{5p+1}{4}-1\right)}{2}=\frac{5p-3}{8} \qquad \ldots(1)$$

Also, product of roots $=\alpha(1+\alpha)=\dfrac{5p}{4}$ $\qquad \ldots(2)$

$\therefore$ From (1) and (2), we get

$$\alpha^2+\alpha=\frac{5p}{4} \Rightarrow \left(\frac{5p-3}{8}\right)^2+\left(\frac{5p-3}{8}\right)=\frac{5p}{4}$$

$\Rightarrow \quad (5p-3)^2+8(5p-3)=80p$

$\Rightarrow \quad 25p^2-70p-15=0$

$\Rightarrow \quad 5p^2-14p-3=0$

$\Rightarrow \quad (5p+1)(p-3)=0$

$\therefore \quad p=\dfrac{-1}{5},3$

Hence, integral value of $p=3$.

5.(7) The equation of the parabola can be taken as

$$y=m\left(x-\frac{1}{4}\right)^2-\frac{9}{8} \quad \text{(given that minimum value is } \frac{-9}{8} \text{ when } x=\frac{1}{4}\text{)}$$

or $\quad y=m\left(x^2+\dfrac{1}{16}-\dfrac{x}{2}\right)-\dfrac{9}{8}$

or $\quad y=mx^2-\dfrac{m}{2}x+\dfrac{m}{16}-\dfrac{9}{8}$ $\qquad \ldots(1)$

Comparing equation (1) with $y=ax^2+bx+c$,

Hence $a=m, b=\dfrac{-m}{2}$ and $c=\dfrac{m}{16}-\dfrac{9}{8}$

But $\quad a+b+c$ is an integer

$\dfrac{m}{2}+\dfrac{m}{16}-\dfrac{9}{8}=\dfrac{9m-18}{16}$ is an integer.

If $\quad a+b+c=0 \Rightarrow m=2$

If $\quad a+b+c=-1 \Rightarrow m=\dfrac{2}{9}$

If $\quad a+b+c=-2 \Rightarrow m=\dfrac{-14}{9}$ which is negative

Hence minimum positive value of coefficient of x^2 which is $m = \frac{2}{9}$

$\Rightarrow \quad p = 2$ and $q = 9$

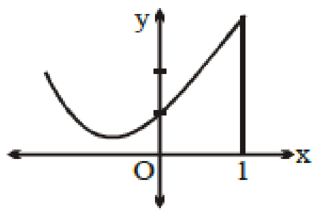
Hence $(q - p) = 7$.

6.(1) Vertex is at $x = b$. Also $f(x)$ is decreasing in $(-\infty, b)$ and increasing in (b, ∞).

$x = b$ is the point of minima.

We have, $f(x) = x^2 - 2bx + 1$

Case I:



Case-I : y = f(x)

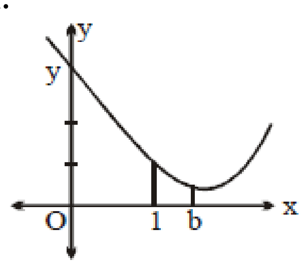
Let $\quad b \le 0$

So, $\quad f(1) - f(0) = 4$

$\Rightarrow \quad (2 - 2b) - 1 = 4$

$\Rightarrow \quad 1 - 2b = 4 \Rightarrow b = \frac{-3}{2}$ (which is < 0)

Case II:



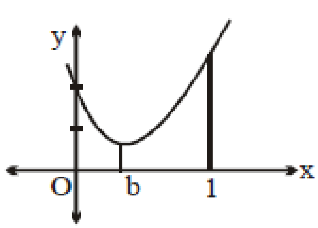
Case-II : y f(x)

Let $b \ge 1$

So, $\quad f(0) - f(1) = 4 \Rightarrow \quad 1 - (2 - 2b) = 4$

$\Rightarrow \quad 2b = 5 \Rightarrow b = \frac{5}{2}$ (which is > 1)

Case III:



Case-III : f(x)

If $\quad 0 < b < 1$

Clearly, max. $[f(0), f(1)] - f(b) = 4$

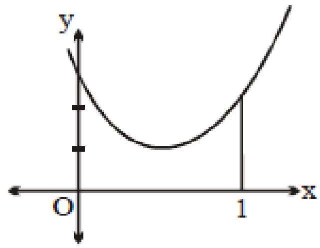
Here, $f(1) > f(0)$

Hence, $f(1) - f(b) = 4$

$\Rightarrow \quad 2-2b-\left(1-b^2\right)=4$

$\Rightarrow \quad b^2-2b+1=4 \Rightarrow \quad b-1=2$ or -2

$\Rightarrow \quad b=3$ or $b=-1$ (Rejected)



Case-III : y = f(x)

Here $\quad f(0)>f(1)$

So, $\quad f(0)-f(b)=4 \Rightarrow \quad 1-\left(1-b^2\right)=4$

$\Rightarrow \quad b=2$ or -2 (Rejected)

Hence $b \in \left\{\frac{-3}{2}, \frac{5}{2}\right\}$

Hence, sum $=\frac{-3}{2}+\frac{5}{2}=1.$